Senior Pre-Calculus (3041)

Lesson for the Week of April 6, 2020 – the concepts contained in this lesson were taught in class but, you may need a refresher, so I included a detailed outline for how to complete each type of problem.

Six Trigonometric Ratios

There are three parts to this lesson:

Part 1: Finding the Six Trig Ratios Given a Triangle

Part 2: Finding the Six Trig Ratios Given a Point on the Terminal Side of an Angle in Standard Position

Part 3: Finding the Six Trig Ratios Given One Trig Ratio

For each section, I give you the **tools** needed, an **example**, and **9 practice problems**. This document is 36 pages. Ten of these pages contains the actual problems that must be completed. Most of this document consist of tools and examples.

There are 3 problem sets each containing 9 problems for a total of 27 problems.

Problems Sets 1, 2 and 3 must be submitted, to me, via email by 3:00 pm on Friday, April 10th.

There is an answer key at the end of the document

If you do not have the ability to submit this assignment via email, please let me know and we will make other arrangements.

These practice problems are mandatory and count towards your Quarter 4 grade.

They will be graded using the following criteria:

- Full Credit
- No Credit – In Progress (which means I will send it back to you to fix)
- No Submission

Please feel free to ask me questions, at any time.

If you want immediate feedback, then contact me during my office hours.

I will have office hours, every day, between the hours of 12:00 pm and 2:00 pm.

Part 1: Finding the Six Trig Ratios Given a Triangle

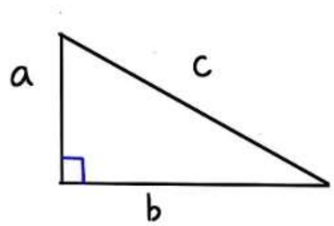
Tools:

Six Trig Ratios with Respect to Opposite, Adjacent, and Hypotenuse

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \text{ctn}\,\theta = \frac{adj}{opp}$$

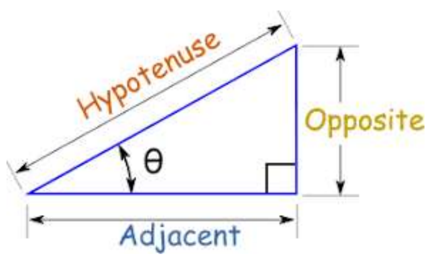
Pythagorean Theorem

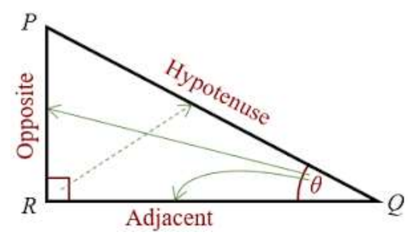


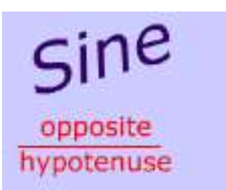
$$c^2 = a^2 + b^2$$

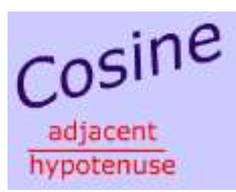
Where c is always the hypotenuse
or
the side opposite the right angle.

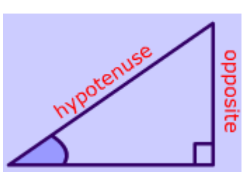
Which Side is Which?

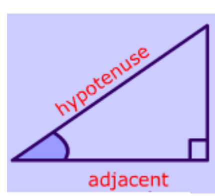


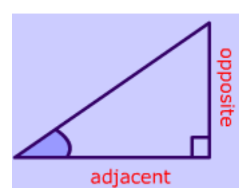


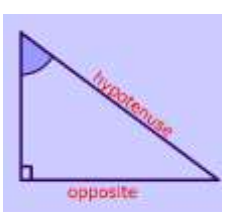


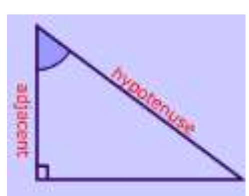


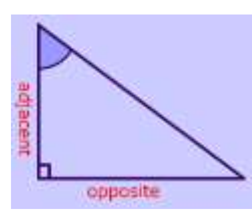






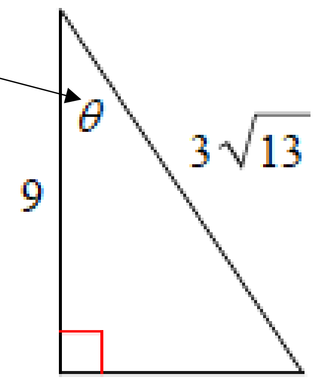






Example 1:

Find the value of the six trig ratios for θ.



Steps:

1. Find the missing side length using the Pythagorean Theorem
2. Identify the side opposite the angle and the side adjacent to the angle
3. Write out the six trig ratios using the values for the "opposite side" the "adjacent side", and the "hypotenuse."
4. Simplify the answers

Step 1: Find the missing side length using the Pythagorean Theorem

$$c^2 = a^2 + b^2$$

$$\left(3\sqrt{13}\right)^2 = a^2 + 9^2$$

$$117 = a^2 + 81$$

When squaring a number like $3\sqrt{13}$ you must square the 3 and the $\sqrt{13}$.

$$\left(3\sqrt{13}\right)^2 = (3)^2 \cdot \left(\sqrt{13}\right)^2$$

$$= 9 \cdot \sqrt{169}$$

$$= 9 \cdot 13$$

$$= 117$$

$$36 = a^2$$

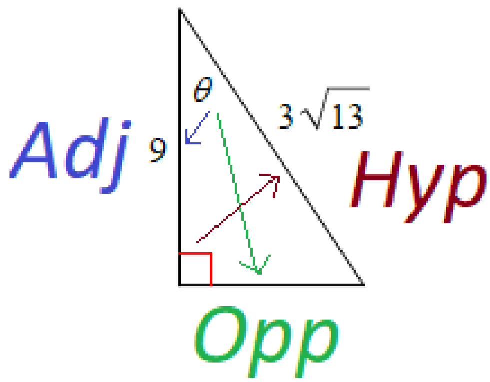
$$\sqrt{36} = \sqrt{a^2}$$

$$\pm 6 = a$$

$$a = 6$$

We want the positive 6 because we are looking for a length.

Step 2: Identify the side opposite the angle and the side adjacent to the angle



Adjacent Side: 9

Opposite: 6

Hypotenuse: $3\sqrt{13}$

Step 3: Write out the six trig ratios using the values for the "opposite side" the "adjacent side", and the "hypotenuse."

Make sure you:

- Use the θ symbol because it is the input part of the function, the ratio is the output part of the function
- Use the equal sign

Adjacent Side: 9 Opposite: 6 Hypotenuse: $3\sqrt{13}$

$$\sin\theta = \frac{opp}{hyp} = \frac{6}{3\sqrt{13}} \qquad \csc\theta = \frac{hyp}{opp} = \frac{3\sqrt{13}}{6}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{9}{3\sqrt{13}} \qquad \sec\theta = \frac{hyp}{adj} = \frac{3\sqrt{13}}{9}$$

$$\tan\theta = \frac{opp}{adj} = \frac{6}{9} \qquad \text{ctn}\,\theta = \frac{adj}{opp} = \frac{9}{6}$$

Step 4: Simplify the answers

$$\sin\theta = \frac{opp}{hyp} = \frac{6}{3\sqrt{13}} = \frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\csc\theta = \frac{hyp}{opp} = \frac{3\sqrt{13}}{6} = \frac{\sqrt{13}}{2}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{9}{3\sqrt{13}} = \frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{3\sqrt{13}}{9} = \frac{\sqrt{13}}{3}$$

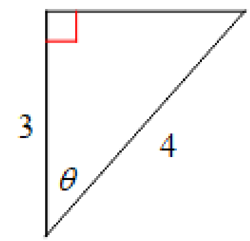
$$\tan\theta = \frac{opp}{adj} = \frac{6}{9} = \frac{2}{3}$$

$$\operatorname{ctn}\theta = \frac{adj}{opp} = \frac{9}{6} = \frac{3}{2}$$

Part 1: Problem Set

Find the value of the six trig ratios for θ.

1.



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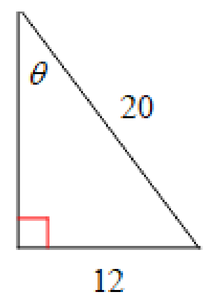

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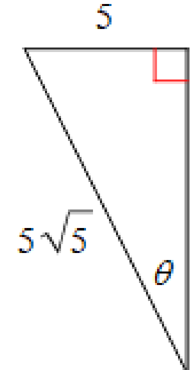

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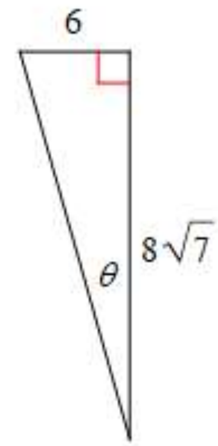

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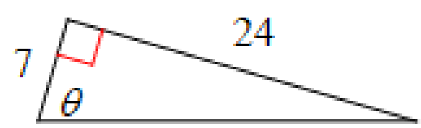

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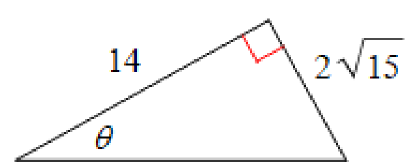

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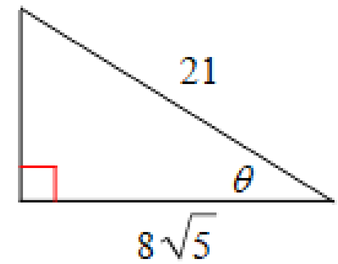

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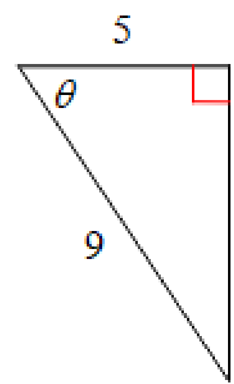

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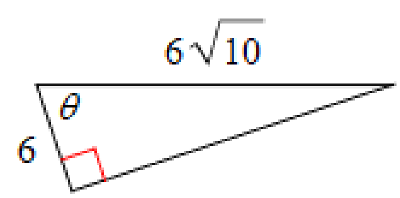


Part 2: Finding the Six Trig Ratios Given a Point on the Terminal Side of an Angle in Standard Position

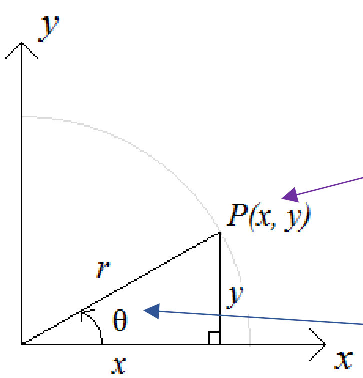
Tools:

Six Trig Ratios with Respect to x, y, and r

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \text{ctn}\,\theta = \frac{x}{y}$$

Pythagorean Theorem



$$r^2 = x^2 + y^2$$

Recall that the equation of a circle and the Pythagorean Theorem are the same thing.

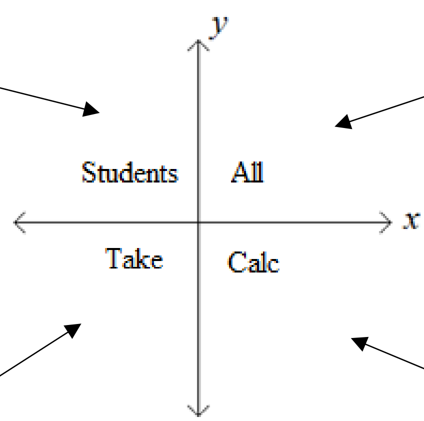
Where r is the radius or the distance from the origin to the point.

This point is on the terminal side of the angle and it is on the circle; this point can be in any quadrant.

This angle is in STANDARD POSITION because the vertex is at the origin and the initial side lies along the positive x-axis.

When the angle is in Quadrant 2, The sine and cosecant values are positive while the other four trig values are negative.

When the angle is in Quadrant 1, all six trig values are positive

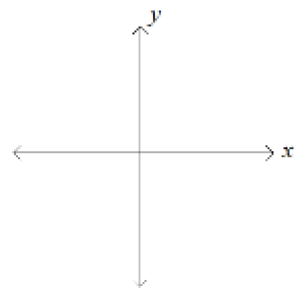


When the angle is in Quadrant 3, The tangent and cotangent values are positive while the other four trig values are negative.

When the angle is in Quadrant 4, The cosine and secant values are positive while the other four trig values are negative.

Example 2:

Find the values of the six trigonometric functions of an angle θ, in standard position, whose terminal side passes through point $P(3, -6)$.



Steps:

1. Graph the angle
2. Identify the x-value and the y-value and use these values to find the length of the radius
3. List the x-value, the y-value, and the r-value for easy reference
4. Write out the six trig ratios using the values of x, y, and r
5. Simplify the answers
6. Check the signs of the answers using "All Student Take Calculus"

Step 1: Graph the angle.

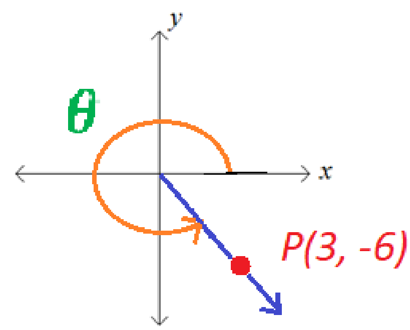
The graph should include the following:

a. The given point with a label

b. A terminal side passing through the given point

c. An arrow indicating the angle; the angle can be positive or negative and can rotate any number of times

d. A label for the angle



Step 2: Find the length of the radius

$$r^2 = x^2 + y^2$$

$$r^2 = (3)^2 + (-6)^2$$

Make sure you square the negative.

$$r^2 = 9 + 36$$

$$r^2 = 45$$

Simplify the Radical.

$$\sqrt{r^2} = \sqrt{45}$$

$$r = \pm 3\sqrt{5}$$

Chose the positive value because the radius is a distance.

$$r = 3\sqrt{5}$$

Step 3: List the x-value, the y-value, and the r-value for easy reference

$x = 3$ $\qquad$ $y = -6$ $\qquad$ $r = 3\sqrt{5}$

Step 4: Write out the six trig ratios using the values of x, y, and r

Make sure you:

- Use the θ symbol because it is the input part of the function, the ratio is the output part of the function
- Use the equal sign

$$\sin\theta = \frac{y}{r} = \frac{-6}{3\sqrt{5}} \qquad \csc\theta = \frac{r}{y} = \frac{3\sqrt{5}}{-6}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{5}} \qquad \sec\theta = \frac{r}{x} = \frac{3\sqrt{5}}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{-6}{3} \qquad \text{ctn}\,\theta = \frac{x}{y} = \frac{3}{-6}$$

Step 5: Simplify the answers

$$\sin\theta = \frac{y}{r} = \frac{-6}{3\sqrt{5}} = \frac{-2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5} = -\frac{2\sqrt{5}}{5} \qquad \csc\theta = \frac{r}{y} = \frac{3\sqrt{5}}{-6} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{5} \qquad \sec\theta = \frac{r}{x} = \frac{3\sqrt{5}}{3} = \sqrt{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-6}{3} = -2 \qquad \text{ctn}\,\theta = \frac{x}{y} = \frac{3}{-6} = -\frac{1}{2}$$

Step 6: Check the signs of the answers using "All Student Take Calculus"

Notice that the angle lies in quadrant 4 so the only two trig function values that are positive are cosine and secant.

$$\sin\theta = \frac{y}{r} = \frac{-6}{3\sqrt{5}} = \frac{-2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5} = -\frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{3\sqrt{5}}{-6} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{3\sqrt{5}}{3} = \sqrt{5}$$

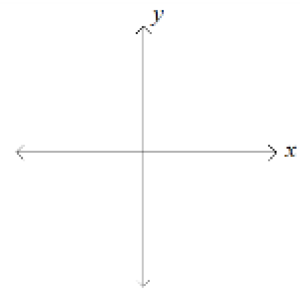
$$\tan\theta = \frac{y}{x} = \frac{-6}{3} = -2$$

$$\operatorname{ctn}\theta = \frac{x}{y} = \frac{3}{-6} = -\frac{1}{2}$$

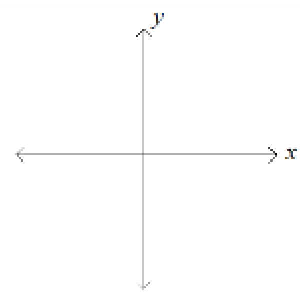
Part 2: Problem Set

Find the values of the six trigonometric functions of an angle θ, in standard position, whose terminal side passes through the given point.

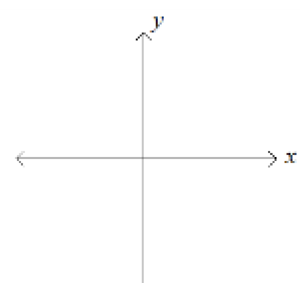
1. $P(-3, -4)$



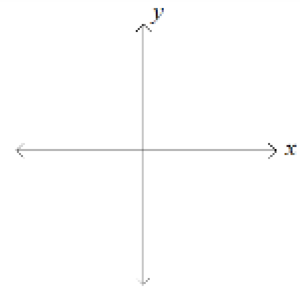
2. $P(\sqrt{17}, -8)$



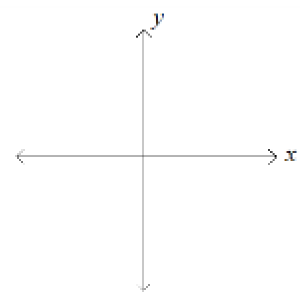
3. $P(-15, 8)$



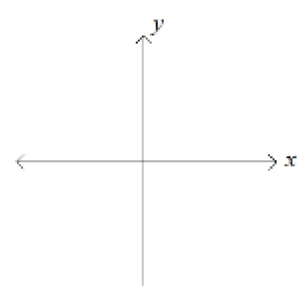
4. $P(15, 5)$



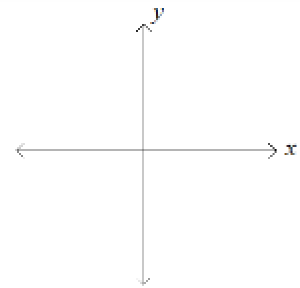
5. $P(-2, -2)$



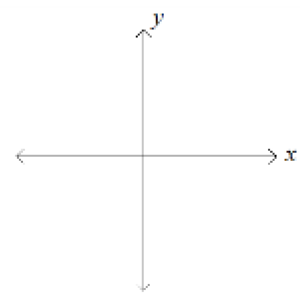
6. $P(\sqrt{15}, 7)$



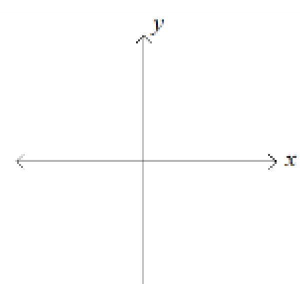
7. $P(16, -14)$



8. $P\left(-8, \sqrt{17}\right)$



9. $P(-3, -20)$



Part 3: Finding the Six Trig Ratios Given One Trig Ratio

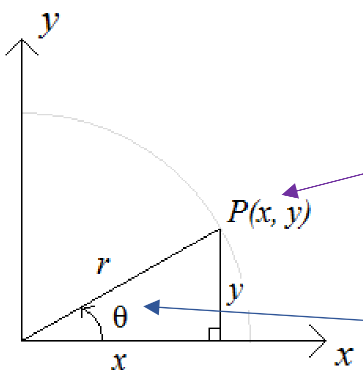
Tools:

Six Trig Ratios with Respect to x, y, and r

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \operatorname{ctn}\theta = \frac{x}{y}$$

Pythagorean Theorem



$$r^2 = x^2 + y^2$$

Recall that the equation of a circle and the Pythagorean Theorem are the same thing.

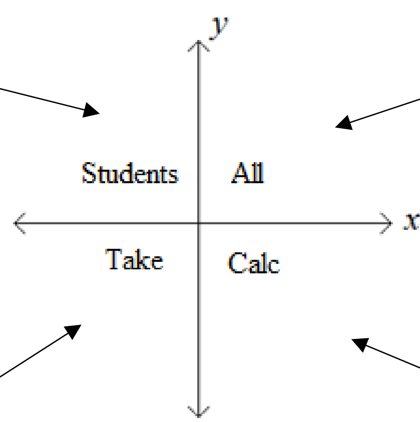
Where r is the radius or the distance from the origin to the point.

This point is on the terminal side of the angle and it is on the circle; this point can be in any quadrant.

This angle is in STANDARD POSITION because the vertex is at the origin and the initial side lies along the positive x-axis.

When the angle is in Quadrant 2, The sine and cosecant values are positive while the other four trig values are negative.

When the angle is in Quadrant 1, all six trig values are positive

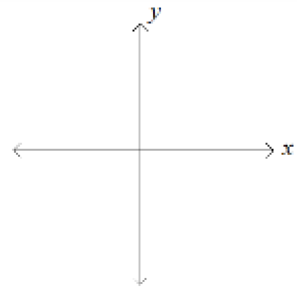


When the angle is in Quadrant 3, The tangent and cotangent values are positive while the other four trig values are negative.

When the angle is in Quadrant 4, The cosine and secant values are positive while the other four trig values are negative.

Example 3:

Using the given information, find the five other trigonometric functions of θ.

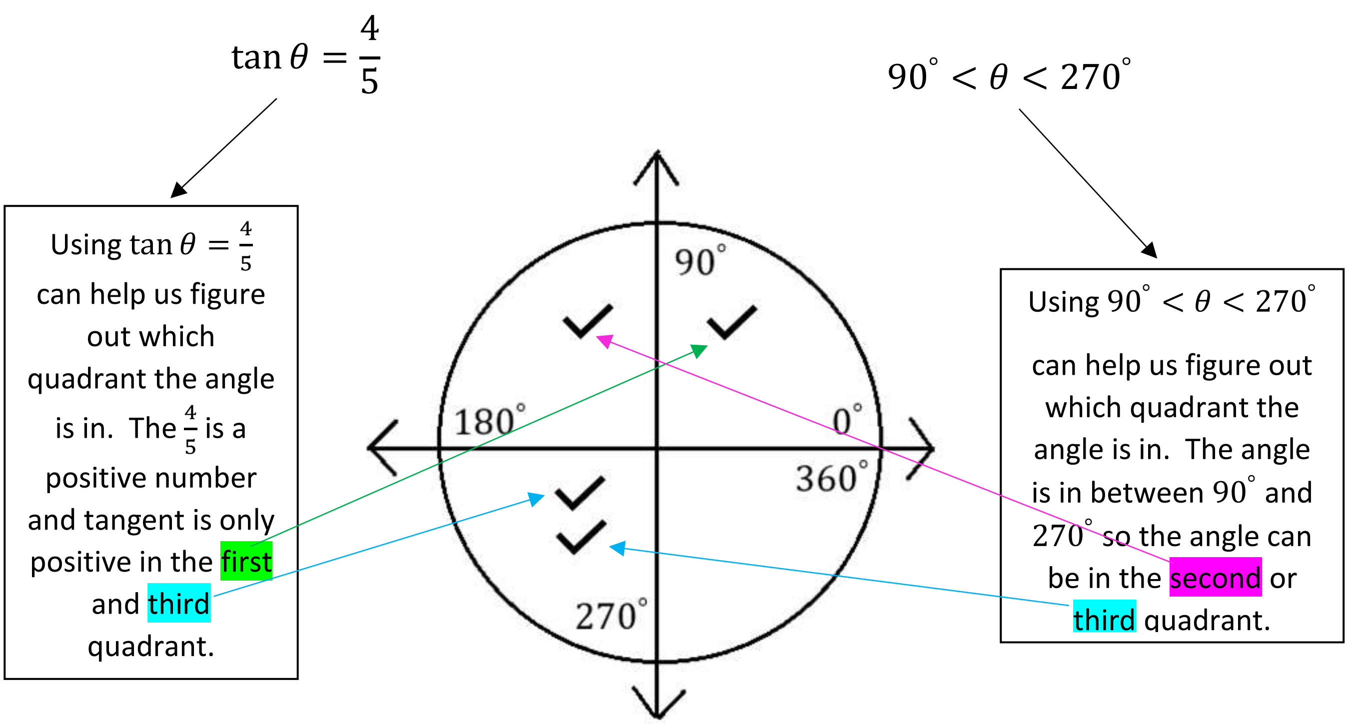


$$\tan\theta = \frac{4}{5}, \quad 90^{\circ} < \theta < 270^{\circ}$$

Steps:

1. Identify which quadrant the angle is in using the given information
2. Graph the angle
3. Identify two of the three values using the given trig function
4. Find the unknown value
5. List the x-value, the y-value, and the r-value for easy reference
6. Write out the six trig ratios using the values of x, y, and r
7. Simplify the answers
8. Check the signs of the answers using "All Student Take Calculus"

Step 1: Identify which quadrant the angle is in using the given information



Notice that the only quadrant that satisfies both conditions is quadrant three… therefore the angle must be in quadrant three. Now we can graph the angle.

Step 2: Graph the angle.

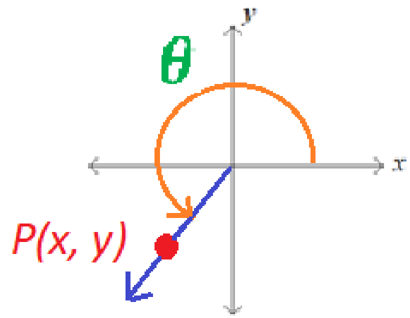
The graph should include the following:

a. The given point with a label

b. A terminal side passing through the given point

c. An arrow indicating the angle; the angle can be positive or negative and can rotate any number of times

d. A label for the angle



Step 3: Identify two of the three values using the given trig function

Using $\tan\theta = \frac{4}{5}$ we can figure out two of the three values that we need to find the six trig functions.

$$\tan\theta = \frac{4}{5}$$

Definition of tangent:

$$\tan\theta = \frac{y}{x}$$

To find the six trig functions, we need the value of x, y, and r.

$$\tan\theta = \frac{y}{x} = \frac{4}{5}$$

therefore, $x = 5$ and $y = 4$.

CAUTION: Be very carefule here! The angle is in the third quadrant and in the third quardrant, the x-value is negative and the y-value is negative. So, $x = -5$ and $y = -4$

Step 4: Find the unknown value

$$r^2 = x^2 + y^2$$

$$r^2 = (-5)^2 + (-4)^2$$

Make sure you square the negative.

$$r^2 = 25 + 16$$

$$r^2 = 41$$

$$\sqrt{r^2} = \sqrt{41}$$

This radical does not simplify.

$$r = \pm\sqrt{41}$$

$$r = \sqrt{41}$$

Chose the positive value because the radius is a distance.

Step 5: List the x-value, the y-value, and the r-value for easy reference

$x = -5$ $\qquad$ $y = -4$ $\qquad$ $r = \sqrt{41}$

Step 6: Write out the six trig ratios using the values of x, y, and r

Make sure you:

- Use the θ symbol because it is the input part of the function, the ratio is the output part of the function
- Use the equal sign

$$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{41}} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{41}}{-4}$$

$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{41}} \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{41}}{-5}$$

$$\tan\theta = \frac{y}{x} = \frac{-4}{-5} \qquad \operatorname{ctn}\theta = \frac{x}{y} = \frac{-5}{-4}$$

Step 7: Simplify the answers

$$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{41}} = \frac{-4}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = -\frac{4\sqrt{41}}{41} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{41}}{-4} = -\frac{\sqrt{41}}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{41}} = \frac{-5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = -\frac{5\sqrt{41}}{41} \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{41}}{-5} = -\frac{\sqrt{41}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-4}{-5} = \frac{4}{5} \qquad \operatorname{ctn}\theta = \frac{x}{y} = \frac{-5}{-4} = \frac{5}{4}$$

Step 8: Check the signs of the answers using "All Student Take Calculus"

Notice that the angle lies in quadrant 3 so the only two trig function values that are positive are tangent and cotangent.

$$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{41}} = \frac{-4}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = -\frac{4\sqrt{41}}{41}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{41}}{-4} = -\frac{\sqrt{41}}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{41}} = \frac{-5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{41}}{-5} = -\frac{\sqrt{41}}{5}$$

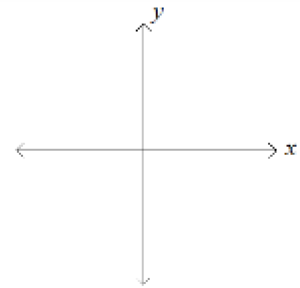
$$\tan\theta = \frac{y}{x} = \frac{-4}{-5} = \frac{4}{5}$$

$$\text{ctn}\,\theta = \frac{x}{y} = \frac{-5}{-4} = \frac{5}{4}$$

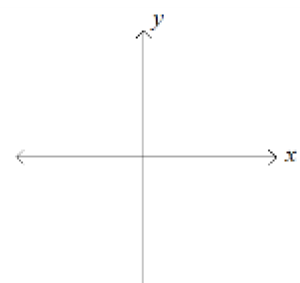
Part 3: Problem Set

Using the given information, find the five other trigonometric functions of θ.

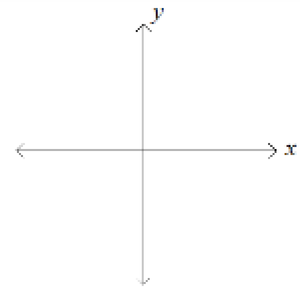
1. $\sin\theta = \frac{3\sqrt{13}}{13}$ and $90^\circ > \theta > 270^\circ$



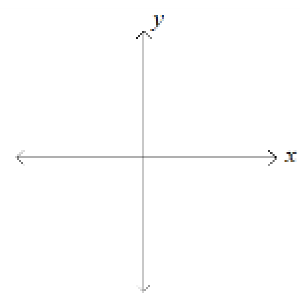
2. $\cos\theta = \frac{1}{5}$ and $90^\circ > \theta > 360^\circ$



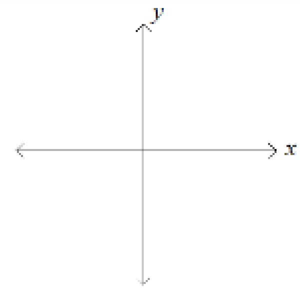
3. $\tan\theta = -\frac{\sqrt{19}}{9}$ and $180^\circ > \theta > 360^\circ$



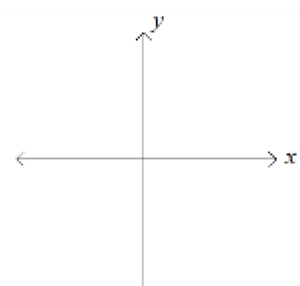
4. $\sec\theta = -\frac{17}{15}$ and $\sin\theta$ is negative



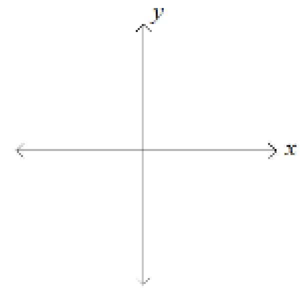
5. $\sin\theta = \frac{2\sqrt{5}}{25}$ and $\tan\theta$ is negative



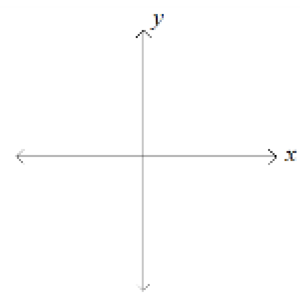
6. $\tan\theta = -\frac{\sqrt{5}}{2}$ and $\cos\theta$ is positive



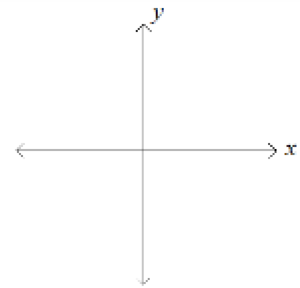
7. $\cot\theta = -1$ and $\sin\theta < 0$



8. $\csc\theta = -\frac{\sqrt{7}}{2}$ and $\tan\theta > 0$



9. $\cos\theta = \frac{9}{10}$ and $\cot\theta < 0$



Part 1: Problem Set

Find the value of the six trig ratios for θ.

$opp = \sqrt{7},\ adj = 3,\ hyp = 4$

1. $c^2 = a^2 + b^2$

$4^2 = a^2 + 3^2$

$16 = a^2 + 9$

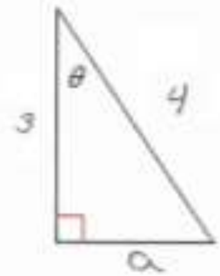
$-9 \quad -9$

$7 = a^2$

$a = \pm\sqrt{7}$

$a = \sqrt{7}$

$\sin\theta = \frac{\sqrt{7}}{4}$	$\csc\theta = \frac{4\sqrt{7}}{7}$
$\cos\theta = \frac{3}{4}$	$\sec\theta = \frac{4}{3}$
$\tan\theta = \frac{\sqrt{7}}{3}$	$\text{ctn}\,\theta = \frac{3\sqrt{7}}{7}$



$opp = 12,\ adj = 16,\ hyp = 20$

2. $c^2 = a^2 + b^2$

$20^2 = a^2 + 12^2$

$400 = a^2 + 144$

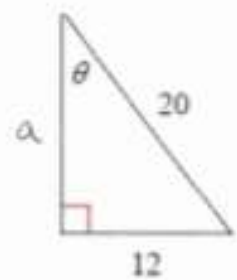
$-144 \quad -144$

$256 = a^2$

$a = \pm 16$

$a = 16$

$\sin\theta = \frac{3}{5}$	$\csc\theta = \frac{5}{3}$
$\cos\theta = \frac{4}{5}$	$\sec\theta = \frac{5}{4}$
$\tan\theta = \frac{3}{4}$	$\text{ctn}\,\theta = \frac{4}{3}$



$opp = 5,\ adj = 10,\ hyp = 5\sqrt{5}$

3. $c^2 = a^2 + b^2$

$(5\sqrt{5})^2 = a^2 + 5^2$

$125 = a^2 + 25$

$-25 \quad -25$

$100 = a^2$

$a = \pm 10$

$a = 10$

$\sin\theta = \frac{\sqrt{5}}{5}$	$\csc\theta = \sqrt{5}$
$\cos\theta = \frac{\sqrt{5}}{2}$	$\sec\theta = \frac{\sqrt{5}}{2}$
$\tan\theta = \frac{1}{2}$	$\text{ctn}\,\theta = 2$

4. $c^2 = a^2 + b^2$

$c^2 = 6^2 + (8\sqrt{7})^2$

$= 36 + 448$

$= 484$

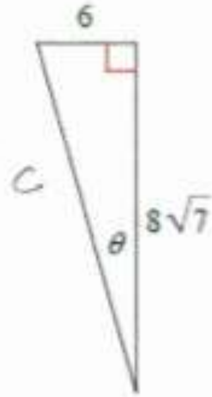
$\sqrt{c^2} = \sqrt{484}$

$c = \pm 22$

$c = 22$

opp = 6, adj = $8\sqrt{7}$, hyp = 22

$\sin\theta = \frac{3}{11}$	$\csc\theta = \frac{11}{3}$
$\cos\theta = \frac{4\sqrt{7}}{11}$	$\sec\theta = \frac{11\sqrt{7}}{28}$
$\tan\theta = \frac{3\sqrt{7}}{28}$	$\text{ctn}\,\theta = \frac{4\sqrt{7}}{3}$



5. $c^2 = a^2 + b^2$

$c^2 = 7^2 + 24^2$

$= 49 + 576$

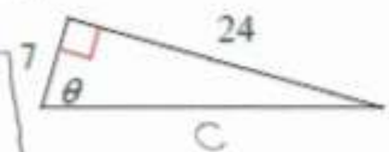
$= 625$

$\sqrt{c^2} = \sqrt{625}$

$c = \pm 25$

$c = 25$

opp = 24, adj = 7, hyp = 25



$\sin\theta = \frac{24}{25}$	$\cos\theta = \frac{25}{24}$
$\cos\theta = \frac{7}{25}$	$\cos\theta = \frac{25}{7}$
$\tan\theta = \frac{24}{7}$	$\text{ctn}\,\theta = \frac{7}{24}$

6. $c^2 = a^2 + b^2$

$c^2 = 14^2 + (2\sqrt{15})^2$

$= 196 + 60$

$= 256$

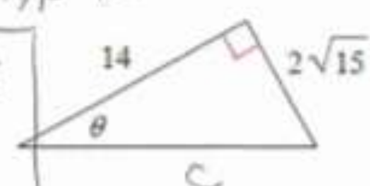
$\sqrt{c^2} = \sqrt{256}$

$c = \pm 16$

$c = 16$

opp = $2\sqrt{15}$, adj = 14, hyp = 16

$\sin\theta = \frac{\sqrt{15}}{8}$	$\csc\theta = \frac{8\sqrt{15}}{15}$
$\cos\theta = \frac{7}{8}$	$\sec\theta = \frac{8}{7}$
$\tan\theta = \frac{\sqrt{15}}{7}$	$\text{ctn}\,\theta = \frac{7\sqrt{15}}{15}$



7. $c^2 = a^2 + b^2$

$21^2 = a^2 + (8\sqrt{5})^2$

$441 = a^2 + 320$

$-320 \quad -320$

$121 = a^2$

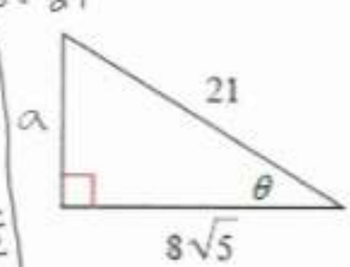
$\sqrt{121} = \sqrt{a^2}$

$a = \pm 11$

$a = 11$

opp $= 11$, adj $= 8\sqrt{5}$, hyp $= 21$

$\sin\theta = \frac{11}{21}$	$\csc\theta = \frac{21}{11}$
$\cos\theta = \frac{8\sqrt{5}}{21}$	$\sec\theta = \frac{21\sqrt{5}}{40}$
$\tan\theta = \frac{11\sqrt{5}}{40}$	$\cot\theta = \frac{8\sqrt{5}}{11}$



8. $c^2 = a^2 + b^2$

$9^2 = a^2 + 5^2$

$81 = a^2 + 25$

$-25 \quad -25$

$56 = a^2$

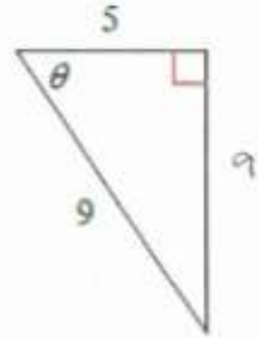
$\sqrt{a^2} = \sqrt{56}$

$a = \pm 2\sqrt{14}$

$a = 2\sqrt{14}$

opp $= 2\sqrt{14}$, adj $= 5$, hyp $= 9$

$\sin\theta = \frac{2\sqrt{14}}{9}$	$\csc\theta = \frac{9\sqrt{14}}{28}$
$\cos\theta = \frac{5}{9}$	$\sec\theta = \frac{9}{5}$
$\tan\theta = \frac{2\sqrt{14}}{5}$	$\cot\theta = \frac{5\sqrt{14}}{28}$



9. $c^2 = a^2 + b^2$

$(6\sqrt{10})^2 = a^2 + 6^2$

$360 = a^2 + 36$

$-36 \quad -36$

$324 = a^2$

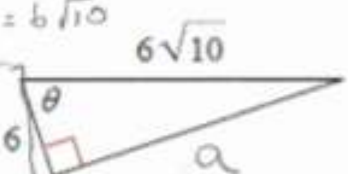
$\sqrt{324} = \sqrt{a^2}$

$a = \pm 18$

$a = 18$

opp $= 18$, adj $= 6$, hyp $= 6\sqrt{10}$

$\sin\theta = \frac{3\sqrt{10}}{10}$	$\csc\theta = \frac{\sqrt{10}}{3}$
$\cos\theta = \frac{\sqrt{10}}{10}$	$\sec\theta = \frac{\sqrt{10}}{3}$
$\tan\theta = 3$	$\cot\theta = \frac{1}{3}$



Part 2: Problem Set

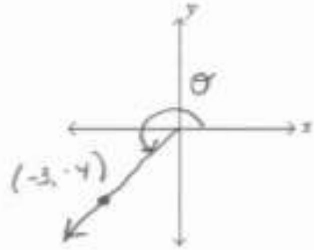
Find the values of the six trigonometric functions of an angle θ, in standard position, whose terminal side passes through point given point.

1. $P(-3,-4)$

$x = -3$, $y = -4$, $r = 5$

$r^2 = x^2 + y^2$
$= (-3)^2 + (-4)^2$
$= 9 + 16$
$r^2 = 25$
$\sqrt{r^2} = \sqrt{25}$
$r = \pm 5$
$r = 5$

$\sin\theta = -\frac{4}{5}$	$\csc\theta = -\frac{5}{4}$
$\cos\theta = -\frac{3}{5}$	$\sec\theta = -\frac{5}{3}$
$\tan\theta = \frac{4}{3}$	$\cot\theta = \frac{3}{4}$

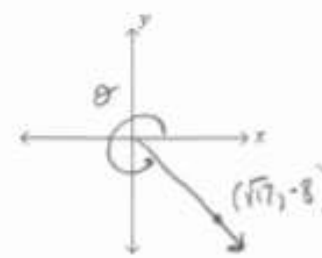


2. $P(\sqrt{17},-8)$

$x = \sqrt{17}$, $y = -8$, $r = 9$

$r^2 = x^2 + y^2$
$= (\sqrt{17})^2 + (-8)^2$
$= 17 + 64$
$r^2 = 81$
$\sqrt{r^2} = \sqrt{81}$
$r = \pm 9$
$r = 9$

$\sin\theta = -\frac{8}{9}$	$\csc\theta = -\frac{9}{8}$
$\cos\theta = \frac{\sqrt{17}}{9}$	$\sec\theta = \frac{9\sqrt{17}}{17}$
$\tan\theta = -\frac{8\sqrt{17}}{17}$	$\cot\theta = -\frac{\sqrt{17}}{8}$

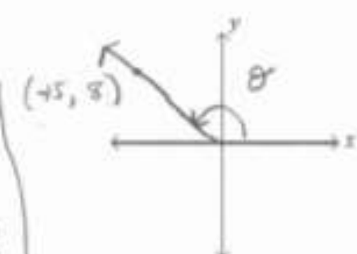


3. $P(-15,8)$

$x = -15$, $y = 8$, $r = 17$

$r^2 = x^2 + y^2$
$r^2 = (-15)^2 + 8^2$
$= 225 + 64$
$r^2 = 289$
$\sqrt{r^2} = \sqrt{289}$
$r = \pm 17$
$r = 17$

$\sin\theta = \frac{8}{17}$	$\csc\theta = \frac{17}{8}$
$\cos\theta = -\frac{15}{17}$	$\sec\theta = -\frac{17}{15}$
$\tan\theta = -\frac{8}{15}$	$\cot\theta = -\frac{15}{8}$



4. $P(15,5)$ $\quad x = 15,\ y = 5,\ r = 5\sqrt{10}$

$r^2 = x^2 + y^2$

$= (15)^2 + (5)^2$

$= 225 + 25$

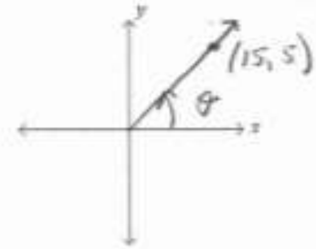
$r^2 = 250$

$\sqrt{r^2} = \sqrt{250}$

$r = \pm 5\sqrt{10}$

$r = 5\sqrt{10}$

$\sin\theta = \dfrac{\sqrt{10}}{10}$	$\csc\theta = \sqrt{10}$
$\cos\theta = \dfrac{3\sqrt{10}}{10}$	$\sec\theta = \dfrac{\sqrt{10}}{3}$
$\tan\theta = \dfrac{1}{3}$	$\text{ctn}\,\theta = 3$



5. $P(-2,-2)$ $\quad x = -2,\ y = -2,\ r = 2\sqrt{2}$

$r^2 = x^2 + y^2$

$= (-2)^2 + (-2)^2$

$= 4 + 4$

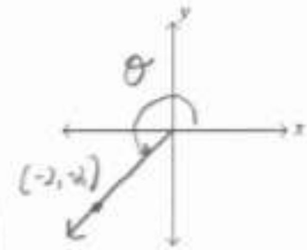
$r^2 = 8$

$\sqrt{r^2} = \sqrt{8}$

$r = \pm 2\sqrt{2}$

$r = 2\sqrt{2}$

$\sin\theta = -\dfrac{\sqrt{2}}{2}$	$\csc\theta = -\sqrt{2}$
$\cos\theta = -\dfrac{\sqrt{2}}{2}$	$\sec\theta = -\sqrt{2}$
$\tan\theta = 1$	$\text{ctn}\,\theta = 1$



6. $P(\sqrt{15},7)$ $\quad x = \sqrt{15},\ y = 7,\ r = 8$

$r^2 = x^2 + y^2$

$= (\sqrt{15})^2 + (7)^2$

$= 15 + 49$

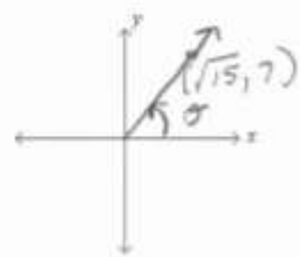
$r^2 = 64$

$\sqrt{r^2} = \sqrt{64}$

$r = \pm 8$

$r = 8$

$\sin\theta = \dfrac{7}{8}$	$\csc\theta = \dfrac{8}{7}$
$\cos\theta = \dfrac{\sqrt{15}}{8}$	$\sec\theta = \dfrac{8\sqrt{15}}{15}$
$\tan\theta = \dfrac{7\sqrt{15}}{15}$	$\text{ctn}\,\theta = \dfrac{\sqrt{15}}{7}$



7. $P(16,-14)$

$x = 16,\quad y = -14,\quad r = 2\sqrt{113}$

$r^2 = x^2 + y^2$

$= (16)^2 + (-14)^2$

$= 256 + 196$

$= 452$

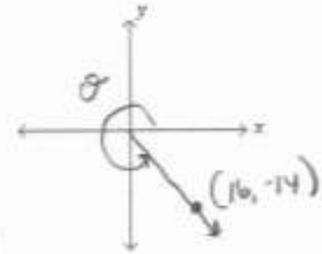
$r^2 = 452$

$\sqrt{r^2} = \sqrt{452}$

$r = \pm 2\sqrt{113}$

$r = 2\sqrt{113}$

$\sin\theta = -\frac{7\sqrt{113}}{113}$	$\csc\theta = -\frac{\sqrt{113}}{7}$
$\cos\theta = \frac{8\sqrt{113}}{113}$	$\sec\theta = \frac{\sqrt{113}}{8}$
$\tan\theta = -\frac{7}{8}$	$\cot\theta = -\frac{8}{7}$



$\frac{-14}{2\sqrt{113}} = \frac{-7}{\sqrt{113}}$

8. $P(-8,\sqrt{17})$

$x = -8,\quad y = \sqrt{17},\quad r = 9$

$r^2 = x^2 + y^2$

$= (-8)^2 + (\sqrt{17})^2$

$= 64 + 17$

$= 81$

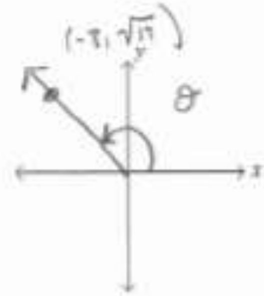
$r^2 = 81$

$\sqrt{r^2} = \sqrt{81}$

$r = \pm 9$

$r = 9$

$\sin\theta = \frac{\sqrt{17}}{9}$	$\csc\theta = \frac{9\sqrt{17}}{17}$
$\cos\theta = -\frac{8}{9}$	$\sec\theta = -\frac{9}{8}$
$\tan\theta = -\frac{\sqrt{17}}{8}$	$\cot\theta = -\frac{8\sqrt{17}}{17}$



9. $P(-3,-20)$

$x = -3,\quad y = -20,\quad r = \sqrt{409}$

$r^2 = x^2 + y^2$

$= (-3)^2 + (-20)^2$

$= 9 + 400$

$= 409$

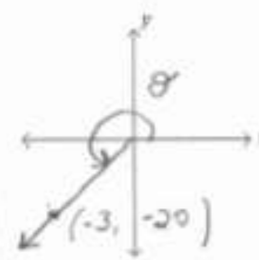
$r^2 = 409$

$\sqrt{r^2} = \sqrt{409}$

$r = \pm\sqrt{409}$

$r = \sqrt{409}$

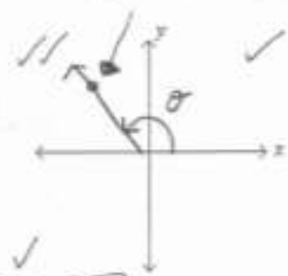
$\sin\theta = -\frac{20\sqrt{409}}{409}$	$\csc\theta = -\frac{\sqrt{409}}{20}$
$\cos\theta = -\frac{3\sqrt{409}}{409}$	$\sec\theta = -\frac{\sqrt{409}}{3}$
$\tan\theta = \frac{20}{3}$	$\cot\theta = \frac{3}{20}$



Part 3: Problem Set

Using the given information, find the five other trigonometric functions of θ. $(-2\sqrt{13}, 3\sqrt{13})$

1. $\sin\theta = \frac{3\sqrt{13}}{13}$ and $90^\circ > \theta > 270^\circ$



Quad 2

$x = -2\sqrt{13},\ y = 3\sqrt{13},\ r = 13$

$\sin\theta = \frac{3\sqrt{13}}{13} = \frac{y}{r}$

$r^2 = x^2 + y^2$

$13^2 = x^2 + (3\sqrt{13})^2$

$169 = x^2 + 117$

$-117 \quad -117$

$52 = x^2$

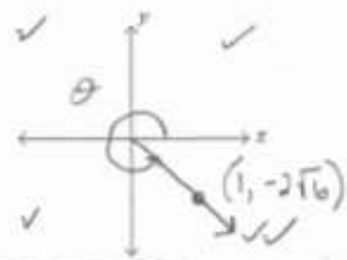
$\sqrt{52} = \sqrt{x^2}$

$x = \pm 2\sqrt{13}$

$x = -2\sqrt{13}$

$\sin\theta = \frac{3\sqrt{13}}{13}$	$\csc\theta = \frac{13\sqrt{13}}{3}$
$\cos\theta = -\frac{2\sqrt{13}}{13}$	$\sec\theta = -\frac{13\sqrt{13}}{2}$
$\tan\theta = -\frac{3}{2}$	$\cot\theta = -\frac{2}{3}$

2. $\cos\theta = \frac{1}{5}$ and $90^\circ > \theta > 360^\circ$



Quad 4

$x = 1,\ y = -2\sqrt{6},\ r = 5$

$\cos\theta = \frac{1}{5} = \frac{x}{r}$

$r^2 = x^2 + y^2$

$5^2 = (1)^2 + y^2$

$25 = 1 + y^2$

$-1 \quad -1$

$24 = y^2$

$y^2 = 24$

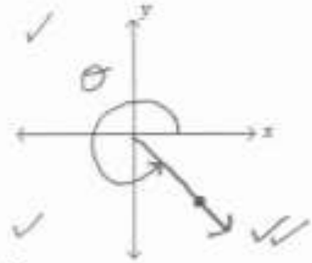
$\sqrt{y^2} = \sqrt{24}$

$y = \pm 2\sqrt{6}$

$y = -2\sqrt{6}$

$\sin\theta = -\frac{2\sqrt{6}}{5}$	$\csc\theta = -\frac{5\sqrt{6}}{12}$
$\cos\theta = \frac{1}{5}$	$\sec\theta = 5$
$\tan\theta = -2\sqrt{6}$	$\cot\theta = -\frac{\sqrt{6}}{12}$

3. $\tan\theta = -\frac{\sqrt{19}}{9}$ and $180^\circ > \theta > 360^\circ$



Quad 4

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{19}}{9}$

$r^2 = x^2 + y^2$

$= (9)^2 + (-\sqrt{19})^2$

$= 81 + 19$

$r^2 = 100$

$\sqrt{r^2} = \sqrt{100}$

$r = \pm 10$

$r = 10$

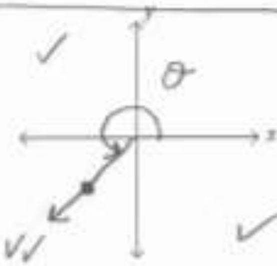
$x = 9$, $y = -\sqrt{19}$, $r = 10$

$\sin\theta = -\frac{\sqrt{19}}{10}$ $\csc\theta = -\frac{10\sqrt{19}}{19}$

$\cos\theta = \frac{9}{10}$ $\sec\theta = \frac{10}{9}$

$\tan\theta = -\frac{\sqrt{19}}{9}$ $\text{ctn}\,\theta = -\frac{9\sqrt{19}}{19}$

4. $\sec\theta = -\frac{17}{15}$ and $\sin\theta$ is negative



Quad 3

$\sec\theta = -\frac{17}{15} = \frac{r}{x}$

$x = -15$, $y = -8$, $r = 17$

$r^2 = x^2 + y^2$

$17^2 = (-15)^2 + y^2$

$289 = 225 + y^2$

$-225 \quad -225$

$64 = y^2$

$\sqrt{y^2} = \sqrt{64}$

$y = \pm 8$

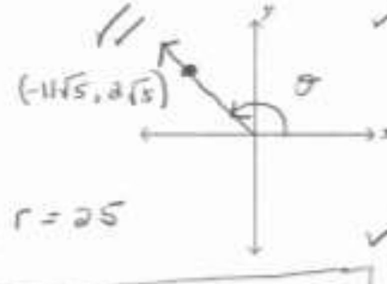
$y = -8$

$\sin\theta = -\frac{8}{17}$ $\csc\theta = -\frac{17}{8}$

$\cos\theta = -\frac{15}{17}$ $\sec\theta = -\frac{17}{15}$

$\tan\theta = \frac{8}{15}$ $\text{ctn}\,\theta = \frac{15}{8}$

5. $\sin\theta = \frac{2\sqrt{5}}{25}$ and $\tan\theta$ is negative



Quad 2

$\sin\theta = \frac{2\sqrt{5}}{25} = \frac{y}{r}$

$x = -11\sqrt{5},\ y = 2\sqrt{5},\ r = 25$

$r^2 = x^2 + y^2$

$25^2 = x^2 + (2\sqrt{5})^2$

$625 = x^2 + 20$

$-20 \quad -20$

$605 = x^2$

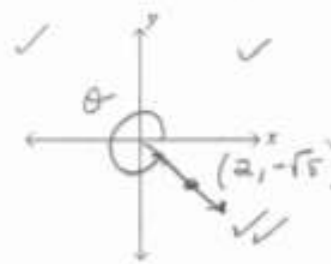
$\sqrt{x^2} = \sqrt{605}$

$x = \pm 11\sqrt{5}$

$x = -11\sqrt{5}$

$\sin\theta = \frac{2\sqrt{5}}{25}$	$\csc\theta = \frac{5\sqrt{5}}{2}$
$\cos\theta = -\frac{11\sqrt{5}}{25}$	$\sec\theta = -\frac{25\sqrt{5}}{55}$
$\tan\theta = -\frac{2}{11}$	$\cot\theta = -\frac{11}{2}$

6. $\tan\theta = -\frac{\sqrt{5}}{2}$ and $\cos\theta$ is positive



Quad 4

$\tan\theta = -\frac{\sqrt{5}}{2} = \frac{y}{x}$

$x = 2,\ y = -\sqrt{5},\ r = 3$

$r^2 = x^2 + y^2$

$= (2)^2 + (-\sqrt{5})^2$

$= 4 + 5$

$= 9$

$r^2 = 9$

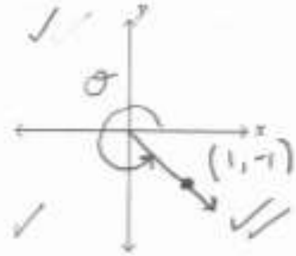
$\sqrt{r^2} = \sqrt{9}$

$r = \pm 3$

$r = 3$

$\sin\theta = -\frac{\sqrt{5}}{3}$	$\csc\theta = -\frac{3\sqrt{5}}{5}$
$\cos\theta = \frac{2}{3}$	$\sec\theta = \frac{3}{2}$
$\tan\theta = -\frac{\sqrt{5}}{2}$	$\cot\theta = -\frac{2\sqrt{5}}{5}$

7. $\cot\theta = -1$ and $\sin\theta < 0$



Quad 4

$\text{ctn}\,\theta = \frac{-1}{1} = \frac{x}{y}$

$x = 1$, $y = -1$, $r = \sqrt{2}$

$r^2 = x^2 + y^2$

$= (1)^2 + (-1)^2$

$= 1 + 1$

$= 2$

$r^2 = 2$

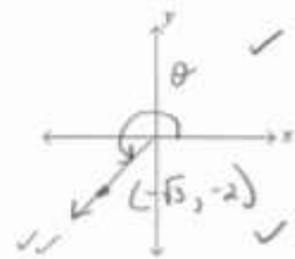
$\sqrt{r^2} = \sqrt{2}$

$r = \pm\sqrt{2}$

$r = \sqrt{2}$

$\sin\theta = -\frac{\sqrt{2}}{2}$	$\csc\theta = -\sqrt{2}$
$\cos\theta = \frac{\sqrt{2}}{2}$	$\sec\theta = \sqrt{2}$
$\tan\theta = -1$	$\text{ctn}\,\theta = -1$

8. $\csc\theta = -\frac{\sqrt{7}}{2}$ and $\tan\theta > 0$



Quad 3

$\csc\theta = -\frac{\sqrt{7}}{2} = \frac{r}{y}$

$x = -\sqrt{3}$, $y = -2$, $r = \sqrt{7}$

$r^2 = x^2 + y^2$

$(\sqrt{7})^2 = x^2 + (-2)^2$

$7 = x^2 + 4$

$-4 \quad\quad -4$

$3 = x^2$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

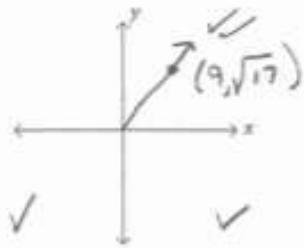
$x = -\sqrt{3}$

$\sin\theta = -\frac{2\sqrt{7}}{7}$	$\csc\theta = -\frac{\sqrt{7}}{2}$
$\cos\theta = -\frac{\sqrt{21}}{7}$	$\sec\theta = -\frac{\sqrt{21}}{3}$
$\tan\theta = \frac{2\sqrt{3}}{3}$	$\text{ctn}\,\theta = \frac{\sqrt{3}}{2}$

9. $\cos\theta = \frac{9}{10}$ and $\cot\theta < 0$

Quad 1

$\cos\theta = \frac{9}{10} = \frac{x}{r}$



$x = 9, \; y = \sqrt{19}, \; r = 10$ ✓ ✓

$r^2 = x^2 + y^2$

$10^2 = 9^2 + y^2$

$100 = 81 + y^2$

$-81 \quad -81$

$19 = y^2$

$\sqrt{y^2} = \sqrt{19}$

$y = \pm\sqrt{19}$

$y = \sqrt{19}$

$\sin\theta = \frac{\sqrt{19}}{10}$	$\csc\theta = \frac{10\sqrt{19}}{19}$
$\cos\theta = \frac{9}{10}$	$\sec\theta = \frac{10}{9}$
$\tan\theta = \frac{\sqrt{19}}{9}$	$\cot\theta = \frac{9\sqrt{19}}{19}$